COLOR IMAGE SEGMENTATION USING MATHEMATICAL MORPHOLOGY

CONTENTS

COLOR IMAGE SEGMENTATION USING MATHEMATICAL MORPHOLOGY
- PRELIMINARY STUDY -

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1. Segmentation in image processing

Image segmentation methods, as well as the associated mathematics models have constantly evolved in the last decades. From the first and very simple image segmentation made through a common threshold we can count, today, over one thousand of algorithms proposed in the literature about segmentation. However, these segmentations algorithms are usually structured around four main steps:

0. The datum of a ***perception space***, which may embed the usual physical space and time, and possibly other feature spaces like histograms, textures or orientations;
1. The choice of a ***criterion*** that translates what we mean by *"homogeneous region"* (in the current case);
2. The ***partitioning*** of the perception space into zones that are homogeneous according the previously defined criterion;
3. The ***maximization*** of all possible partitions.

Because in some cases like microscopy, human perception may be totally meaningless while image processing demands quantitative analysis, we can skip the first ***"0"*** step and describe the segmentation algorithm from the next step: the choice of a criterion.

1.1. The 1st step: the definition of a criterion

We choose to classify the pixels according to the criterion of ***flat zones*** that we had already in mind (for other cases, color, shape, or other criteria could be more convenient).
Criterion definition: Given a class of Φ functions from set **E** into set **T**, a criterion σ is a ***binary function*** $\sigma:\Phi\otimes\Pi(E)\rightarrow[0,1]$ defined like:

$\sigma\,[f,A] = 1$ when the criterion is satisfied over **A**

$\sigma\,[f,A] = 0$ when not,

$\forall f\in\Phi,\ \forall A\subseteq E.$

Examples:

- The flat zones of function f (i.e. the zones where the function is constant)
- A threshold (i.e. the zones where the function is above a given value)

1.2. The 2nd step: the definition of a partition

In the second step we replace the pixels by a partition of the space into regions or classes.

Partition definition: A partition of space **E** is a mapping $D:E\rightarrow\Pi(E)$ that associates with each pixel x to the class $D(x)$ it belongs, such that:

(i) the space is covered: $\forall\, x\in E \Rightarrow x\in D(x)$

(ii) there is no overlapping: for all $(x, y)\in E$

either $D(x) = D(y)$
or $D(x) \cap D(y) = \{\varnothing\}$

1.3. The 3rd step: the maximum partition

In the third step we search among all partitions of the space into regions that fulfill the "flat zones" criterion, if there is a larger one satisfying this criterion. First of all, we must determine if such a largest partition always exists. That means that we must answer to the following two questions:

1. *Does the "the largest partition" of a family exists?* The answer is positive because the partitions of a set **E** do form a complete lattice.
2. *Is the largest partition of a family an element of the defined family?* The answer is uncertain and we must determine later by further computing methods.

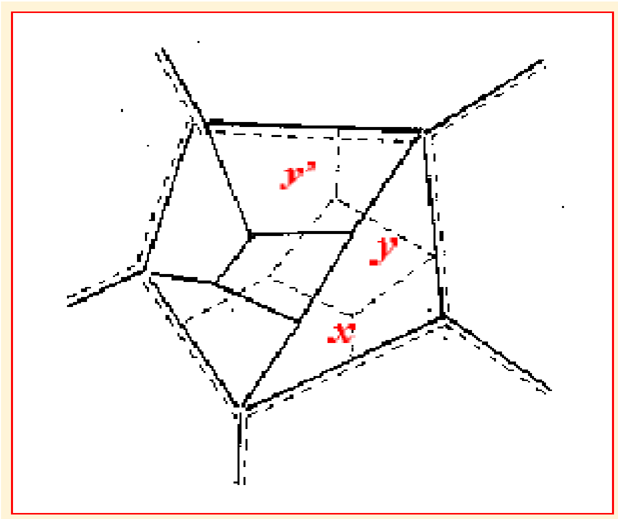
Partition Lattice

We can say that a partition is larger, or is the largest one, because of the:

Proposition 1: The partitions of **E** form a ***complete lattice*** Δ for the ordering relationship defined as:

- Partition **D** is smaller than **D'** (**D** < **D'**) when each class of **D** is included in a class of **D'**.

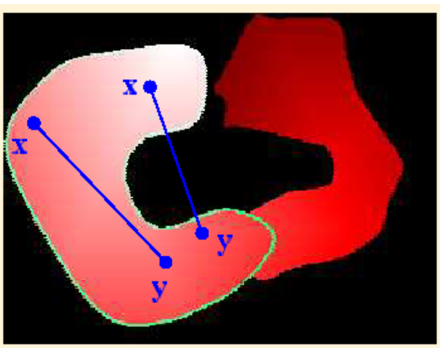
Prof: Indeed, ∀ **D, D'** partitions of **E** they can be compared by the above ordering relationship and also the largest element of **D** is **E** itself and the smallest one is the decomposing of **E** into all its points.



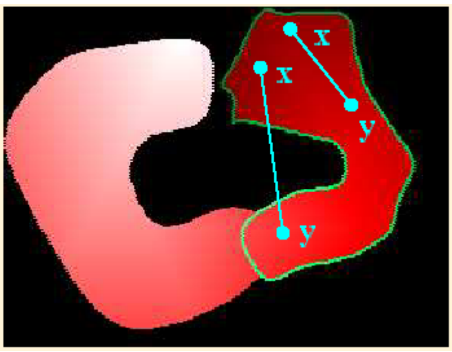
The ***sup*** is the pentagon with common boundaries. The ***inf***, simpler, is obtained by intersecting the cells.

The answer to the second question cannot be always positive as we can see in the next two contra examples:

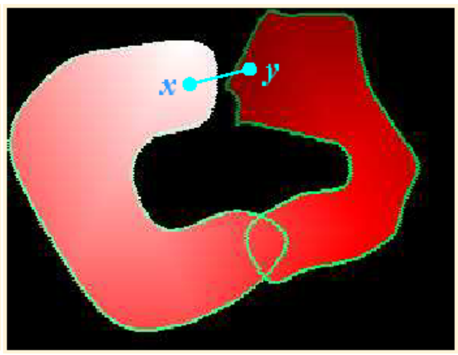
a) Lipschitz function definition: A function f is ***Lipschitz*** of parameter k when $|f(x)-f(y)| \leq k\, \mathbf{d}(x,y)$.



This function is ***Lipschitz*** in the ***left*** part

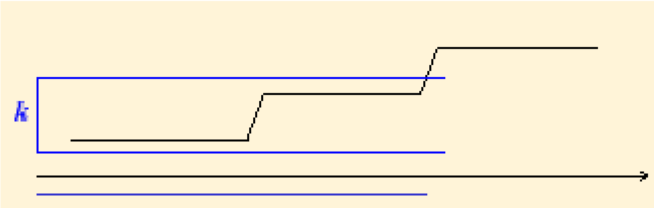


This function is ***Lipschitz*** in both ***left*** and ***right*** part

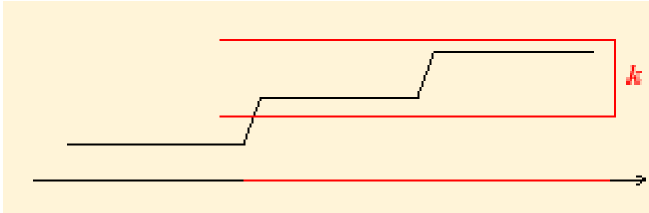


This function is ***Lipschitz*** in both ***left*** and ***right*** part but not inside both parts.
Therefore there is no largest partition!

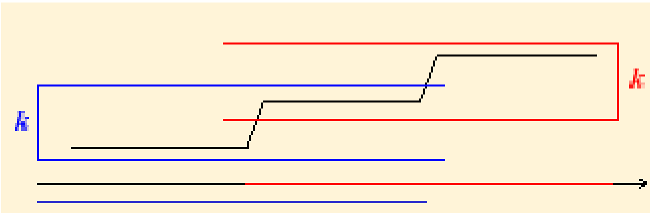
b) Thick flat zones case. Definition: $\forall\, x, y \in A \Rightarrow |f(x)-f(y)| \leq k$



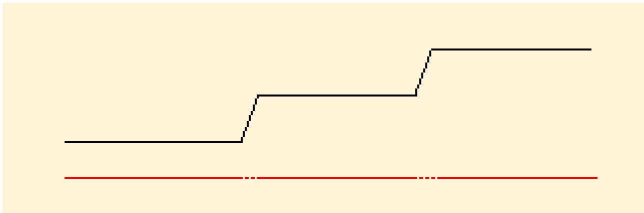
b1)
The ***blue*** segment fulfils the criterion



b2)
The ***blue*** segment fulfils the criterion
The ***red*** segment also fulfils the criterion



b3)
The ***blue*** segment fulfils the criterion
The ***red*** segment also fulfils the criterion
But their union does not fulfil the criterion.
Therefore there is no largest partition!



<u>Thin flat zones case</u>
However, for $k=0$, we find the usual flat zones :
$\forall\, x, y \in A \Rightarrow |f(x)-f(y)| \leq 0$
Then the ***largest partition exists*** and is composed of the ***red segments*** of the constant regions or the ***points***, elsewhere.

2. <u>Connection</u>

<u>Definition</u>: Let **E** be an arbitrary space. We call ***connected class***, or ***connection*** $\mathcal{X}$ any family in $\Pi(\mathbf{E})$ such that:

i) $\{\varnothing\} \in \mathcal{X}$;

ii) $\forall\, x \in \mathbf{E} \Rightarrow \{x\} \in \mathcal{X}$;

(class $\mathcal{X}$ contains always the singletons, plus the empty set)

iii) $\forall \{\mathbf{A_i}\}, \mathbf{A_i} \in \mathcal{X} : \{\cap \mathbf{A_i} \neq \varnothing\} \Rightarrow \{\cup \mathbf{A_i} \in \mathcal{X}\}$;

(the union of elements of $\mathcal{X}$ whose intersection is not empty is still in $\mathcal{X}$)

The elements $\mathbf{C} \in \mathcal{X}$, are said to be ***connected***.
Although such a definition does not involve any topological background, both ***topological*** and ***arcwise*** connectivities are particular connections.

2.1. <u>Point connected opening</u>

Given a set **A** and a point $x \in \mathbf{A}$, we consider the union $\gamma_x(\mathbf{A})$ of all connected components containing x and included in **A**

$$\gamma_x(\mathbf{A}) = \cup\{\mathbf{C} \mid \mathbf{C} \in \mathcal{X},\, x \in \mathbf{C} \subseteq \mathbf{A}\} \qquad (1)$$

<u>Theorem of the point connected opening:</u> The family $\{\gamma_x, x \in \mathbf{E}\}$ is made of openings, called ***point connected opening***, such that:

i) $\gamma_x(x)=\{x\}$, $x \in \mathbf{E}$
ii) $\gamma_y(\mathbf{A})$ and $\gamma_z(\mathbf{A})$ $y,z \in \mathbf{E}$, $\mathbf{A} \subseteq \mathbf{E}$ are disjoint or equal
iii) $x \notin \mathbf{A} \Rightarrow \gamma_x(\mathbf{A}) = \{\varnothing\}$

and the datum of a connected class $\mathcal{X}$ on $\Pi(\mathbf{E})$ is equivalent to such a family.

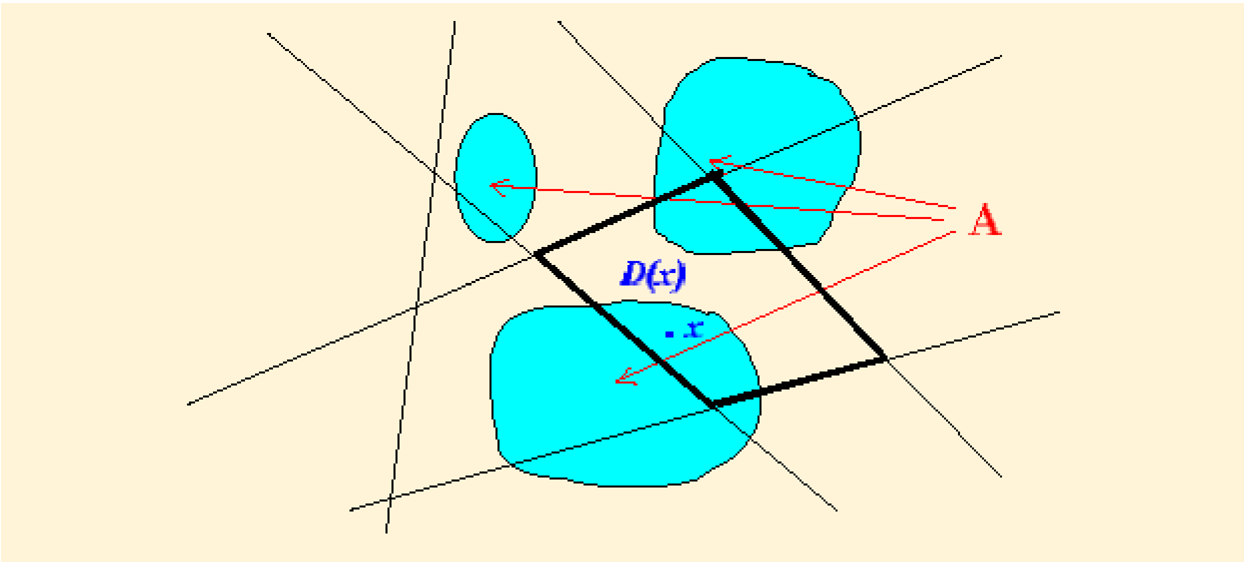
In other words, every $\mathcal{X}$ induces a unique family of openings satisfying ***i)*** to ***iii)***, and the elements of $\mathcal{X}$ are the invariant sets of the said family $\{\gamma_x, x \in \mathbf{E}\}$.

2.2. Connection by partitioning

Given a partition **D** of space **E**, all subsets of all classes $\{\mathbf{D}(x), x\in\mathbf{E}\}$ form a family closed under union. Hence we have the connection:

$$\mathcal{X} = \{\mathbf{A} \cap \mathbf{D}(x),\ x\in\mathbf{E},\ \mathbf{A}\in\Pi(\mathbf{E})\}$$

The connected component $\gamma_x(\mathbf{A})$, $x\in\mathbf{E}$, equals the intersection $\mathbf{A} \cap \mathbf{D}(x)$ between **A** and the class of the partition at point x.



2.3. Properties of the connections. The partitioning theorem

Arc generalization: Set **X** is $\mathcal{X}$-connected iff for all points y and z of **X** we can find a $\mathcal{X}$-component **Y** included in **X** and that contains y and z.

Connection partitioning theorem: Let $\mathcal{X}$ be a connection on $\Pi(\mathbf{E})$. For each set $\mathbf{A}\in\Pi(\mathbf{E})$ the maximal connected components included in **A,** partition **A** into its connected components. This partition is increasing in that if $\mathbf{A}\subset\mathbf{A'}$, then any connected component of **A** is upper bounded by a connected component of **A'**.

Lattice of the connections: The set of the all connections on $\Pi(\mathbf{E})$ is closed under intersection; it is thus a complete lattice in which the supremum of a family $\{\mathcal{X}_i\,;\ i\in \mathbf{I}\}$ is the least connection containing $\cup\mathcal{X}_i$,

$$\mathbf{inf}\,\{\mathcal{X}_i\} = \cap\mathcal{X}_i, \quad \text{and} \quad \mathbf{sup}\,\{\mathcal{X}_i\} = \mathcal{X}\{\cup\mathcal{X}_i\}$$

Proof: Let's start from the point opening defined in equation (1) $\gamma_x(\mathbf{A}) = \cup\,\{\,\mathbf{C}\,|\,\mathbf{C}\in\mathcal{X}\,, x\in\mathbf{C}\subseteq\mathbf{A}\,\}$

1) As point x spans set **E**, we have, for all $\mathbf{A}\in\Pi(\mathbf{E})$

$$\cup_{x\in\mathbf{E}}\,[\,\gamma_x(\mathbf{A})] \supseteq \cup_{x\in\mathbf{E}}\,\cup_{a\in\mathbf{E}}\,[\,\gamma_x(a)] = \cup_{a\in\mathbf{E}}\,\cup_{x\in\mathbf{E}}\,[\,\gamma_x(a)] = \mathbf{A} \qquad \textbf{(2)}$$

Hence

$$\cup_{x\in\mathbf{E}}\,[\,\gamma_x(\mathbf{A})] = \mathbf{A} \qquad \textbf{(3)}$$

so that the $\gamma_x(\mathbf{A})$ partition **A.**

2) Consider another partition of **A** into $\mathbf{A'}_j\in\mathcal{X}$, each x belongs to one $\mathbf{A}_i$ and one $\mathbf{A'}_j$, hence to $\mathbf{A}_i\cap\mathbf{A'}_j$, so that $\mathbf{A}_i\cup\mathbf{A'}_j$ is connected.

It follows from **(1)** that $\mathbf{A}_i \supseteq \mathbf{A}_i\cup\mathbf{A'}_j$, hence $\mathbf{A'}_j \subseteq \mathbf{A}$, therefore the $\gamma_x(\mathbf{A})$ produce the largest connected partition of **A.**

3) This partition is increasing because if x=$\mathbf{A}_i$=$\gamma_x(\mathbf{A})$ then $\mathbf{A} \subseteq \mathbf{B}$, implies $x\in\gamma_x(\mathbf{A}) \subseteq \gamma_x(\mathbf{B})$ but is precisely a connected component of **B**.

3. Segmentation

Let **E** and **T** be two arbitrary sets. Let class Φ be a family of functions $f : \mathbf{E} \to \mathbf{T}$, and be a criterion.

Given a function $f\in\Phi$ let $\{\mathbf{D}_i\,(f)\}$ be the family of all partitions of set **E** into homogenous zones of f according to criterion σ.

Definition of segmentation: We say that criterion ***σ segments*** class **Φ** when for each function $f \in \Phi$, with property $\sigma[f,\{x\}]=1$, $\forall\, x \in E$, the family $\{D_i(f)\}$ admits a supremum $\vee\{D_i(f)\}$. Then the partition $\vee\{D_i(f)\}$ defines the ***segmentation of f*** with respect to ***σ***.

The ***segmentation*** $D = \vee\{D_i(f)\}$ decomposes set **E** into zones that are:

- disjoint and cover the whole space **E**
- where function ***f*** is homogeneous according to criterion ***σ***
- and where the class of the partition at each point is the largest possible one that satisfies criterion ***σ***

For using this notion in practice, we need a theorem that links the concept of a segmentation with tools we can handle, unless we accept to check all possible partitions for each function ***f***.

The convenient notion turns out to be that of a ***connection***.

3.1. Connective criterion

It remains to introduce the last piece of the puzzle, namely the following connection property for criteria.

Connective criterion: A criterion ***σ*** is connective when for any family $\{A_i\}$ and any function $f \in \Phi$ we have:

1) $\sigma[f,\{x\}]=1$ for all x ;

2) $\mathrm{I}\, A_i \neq \{\varnothing\}$ and $\sigma[f, A_i]=1 \Rightarrow \sigma[f, \mathrm{Y} A_i]=1$

More explicit we can say when ***f*** satisfies ***σ*** on **A** and on **B**, and when **A** and **B** have at least one common point, then ***f*** satisfies ***σ*** on **A Y B**.

Observation: Flat zones, zones above a given threshold, etc. are connective, but ***Lipschitz*** criterion is not connective.

3.2. The segmentation theorem

Theorem: Given a criterion ***σ*** on **Φ⊗Π(E)**, the three following statements are equivalent:

1) Criterion a is connective;

2) Given $f \in \Phi$, the class of those sets on which criterion a is satisfied forms a connection **X;**

3) Criterion ***σ*** segments all functions $f \in \Phi$.

Observations:

- ➢ The concept of a connection is exactly right for the theorem to work.
- ➢ If set **E** was not previously provided with a connection, then criterion ***σ*** provides **E** with a connection, X say.
- ➢ Conversely, if space **E** was initially provided with connection **X'** then the intersection **X∩X'** generates the maximum partition for the intersection of the two constraints.

3.3. Comments on segmentation theorem

- ➢ The theorem links segmentation with the connective property of the criterion, that we can more easily handle (1^{st} —> 2^{nd} moment)
- ➢ Remarkably, it is possible to identify the notion of segmentation with some families of connections without having equipped neither the starting set **E**, nor the arrival one **T**, with any property.
- ➢ Indeed, theorem opens the way to all applications where heterogeneous variables are defined over the space. This circumstances arise for example:

○ in color imagery, with the hue, or

○ in geography, where radiometric data (satellite images) live together with physical ones (altitude, slope of the ground, sunshine, distance to the sea, etc.) and with statistical data (demography, fortunes, diseases, etc.).

➢ It will be possible to make precise the segmentation theorem
  ○ by classifying the connective criteria, and by giving a few examples of them
  ○ by analyzing the interactions between working field and segmented objects and
  ○ above all by listing and describing the various techniques it provides for combining criteria, namely :
    - ***intersection and union*** of criteria (infinimum and supremum in the convenient lattice);
    - ***ordered segmentations*** by multiple criteria;
    - ***composition products*** (connected operators).

3.4. Connected operators

Suppose that a first mapping $\psi:\Phi\rightarrow\Phi$ acts on function f prior to its segmentation. Then the pair $\{\sigma, \psi\}$, considered as a whole, ***defines the criterion***

$$\sigma[\psi(f),A] = 1 \text{ or } 0.$$

Property: σ connective $\Rightarrow$ $\{\sigma, \psi\}$ connective as($\psi(\Phi)\subseteq\Phi$).

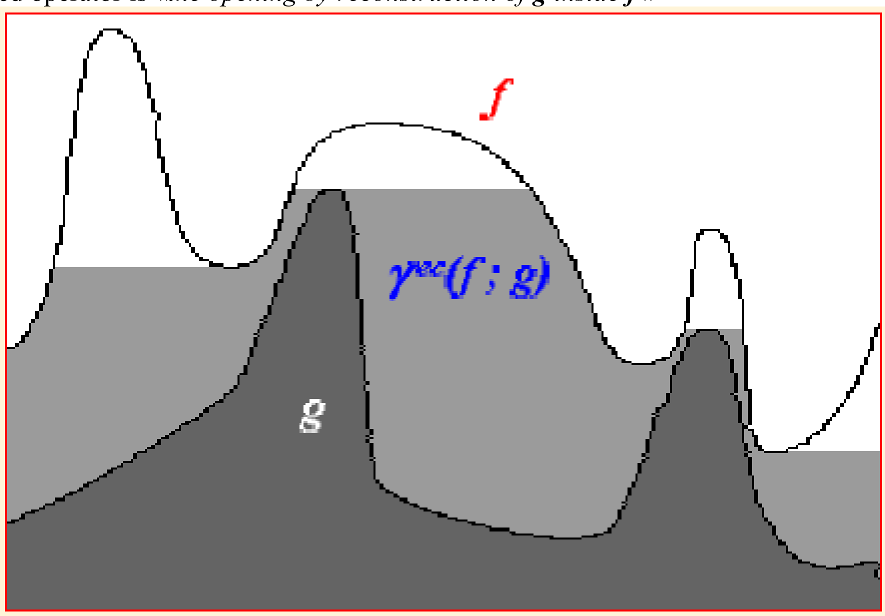
Definition: Operator ψ is said to be ***connected*** when

$$\sigma[f,A] = 1 \quad\Rightarrow\quad \sigma[\psi(f),A] = 1$$

Property: when ψ is ***connected***, the segmentation partition of f according to {a, iff}, is larger than that by σ (some classes are clustered).

3.5. Opening by reconstruction

The basic connected operator is «*the opening by reconstruction of* ***g*** *inside* ***f***»



3.6. Hierarchies of connected operators

Increasing semi-groups. Let ψ and ψ' be two connected operators. We have

$$\sigma[f,A] = 1 \quad\Rightarrow\quad \sigma[\psi(f),A] = 1 \quad\Rightarrow\quad \sigma[\psi'\psi(f),A] = 1$$

i.e. $\psi'\psi$ is connected, and the connection $(\sigma, \psi'\psi)$ contains (σ, ψ).

This suggests to introduce the following ***increasing semi-groups*** $\{\psi_\lambda, \lambda\geq 0\}$ where the product $\psi_\nu=\psi_\lambda\psi_\mu$ acts more than each of its factors, i.e. such that

1) $\forall\, \lambda, \mu\geq 0 \quad\Rightarrow\quad \nu\geq \sup(\lambda, \mu)$

2) $\forall\ \nu \geq \lambda \geq 0$ there exists μ with $\nu \geq \mu \geq 0$ such that $\psi_\nu = \psi_\lambda \psi_\mu$

Property: A semi-group of connected operators is increasing iff the family $\{(\sigma,\psi_\lambda),\ \lambda \geq 0\}$ is totally ordered in the lattice of the connective criteria (or of the corresponding connections).

Then the segmentations of function f by the (σ,ψ_λ) ***increase*** with λ and the set of the contours ***decreases***.

3.7. Examples

Band segmentation intersection

a) ***Original HSV image*** — ***b)*** ***Hue band segmentation*** — ***c)*** ***Value band segmentation*** — ***d)*** ***Hue -Value intersection***

Hierarchical lasso

Goal: To adapt an rough manual contouring to the actual contours of the object.
Implementation: Take the union of the largest classes of the hierarchy that are inside the manual contour.

a) ***Rough manual contour*** — ***b)*** ***Result by hierarchy*** — ***c)*** ***Result by Photo Paint***

Magic wand

Goal: To extract a region of uniform color..
Implementation: In the hierarchy, take the largest class at point x whose average color lies between given bounds.

a)
Original image

b)
Hierarchical magic wand

c)
Photo Paint magic wand

4. Color image segmentation

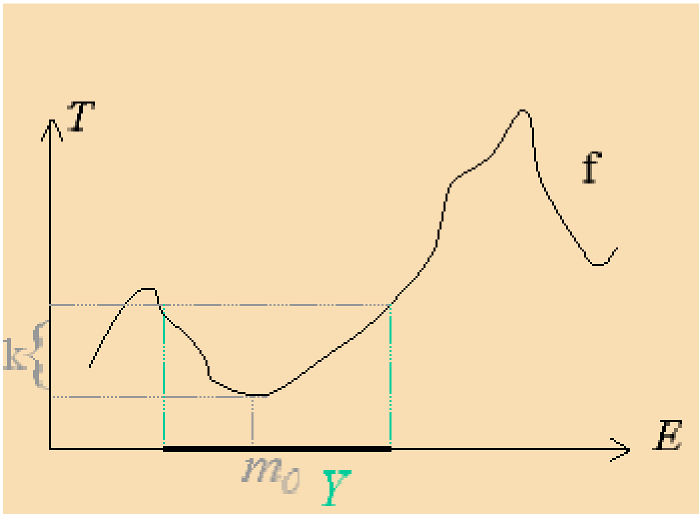
4.1. Color jump connection

Jump connection: In the case of color images the set **E** represents P^n, provided with the **arcwise** connection, and function f : **E** → **T** is fixed. The class $X \in \Pi(P^n)$ which is composed of

i) the singletons plus the empty set,

ii) all connected sets around each minimum and where the value of f is less than k above the minimum,

forms a second connection on $\Pi(P^n)$, called "***jump connection from minima***"(respectively ***maxima***)
One can combine the two connections from maxima and minima



A connected component in the jump connection of range k from the minima

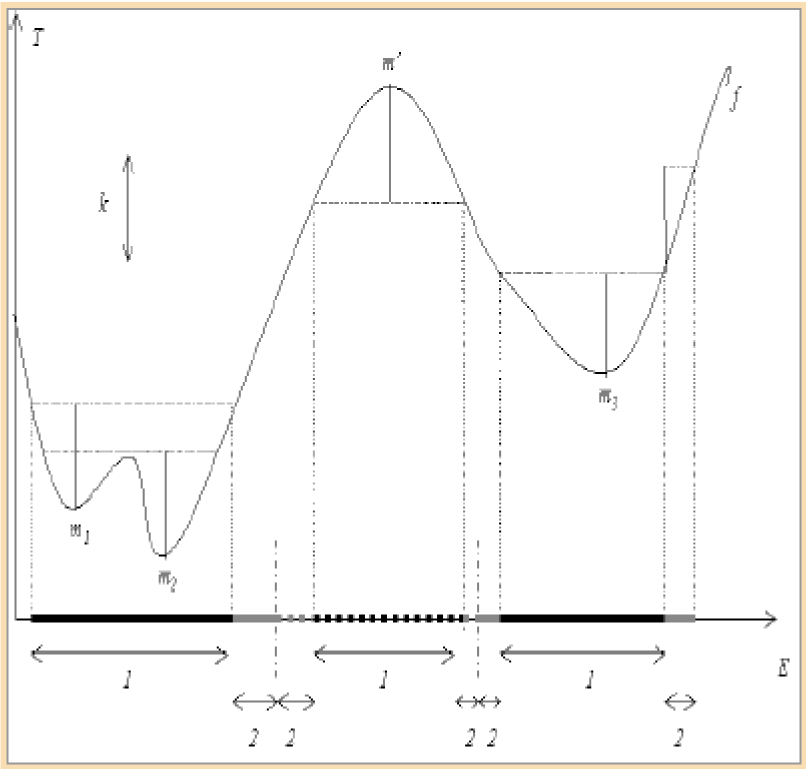
4.2. Iterated jump connection

Such iterations result in a an ***optimal mixed*** segmentation. The algorithm for implementation of this segmentation can be described as:

1. Segment f by a ***jump connection*** and let be S_1 the singleton zone.

2. Restrict f to $S_1 \Rightarrow$ the result function f_1 will be subsequently segmented according to the same jump connection, hence S_2, and f_2, etc...

3. For the sake of self duality, we can use both extrema and progress upwards and downwards.



Iterated jump connection

4.3. Color gradients

In P^n, in order to determine the modulus of the gradient, at point x, of a differentiable function f we use:

$$(\mathbf{incr}\, f)(x) = \vee[\{|f(x) - f(y)|,\ y \in \mathbf{B}_r(x)\}] / \mathbf{r}$$

where $\mathbf{B}_r(x)$ is a small ball centered at x with radius r. The gradient is then the limit, denoted $\alpha(f)$, of $\mathbf{incr}\, f$ as $r \to 0$.

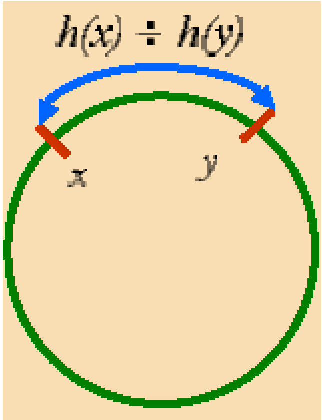
By introducing the ***Minkowski dilation*** δ_r and ***erosion*** ε_r by $\mathbf{B}_r$ we obtain the equivalent definition

$$\mathbf{incr}\, f = [(\delta_r(f) - f) \vee (f - \varepsilon_r(f))] / \mathbf{r}$$

In the digital 2-D space Z^2, this last relation gives $\alpha(f)$ by taking $\mathbf{r} = 1$ and by replacing $\mathbf{B}_r$ by the unit square or hexagon **K**.

4.4. Gradients with values on X

Let h : **E** →X be an angular function, such as the ***hue***. As the definition of the gradient involves increments only, it can be transposed to h by replacing $|h(x) - h(y)|$ by the acute angle $|h(x) \div h(y)|$.



This leads to the new expression definition:

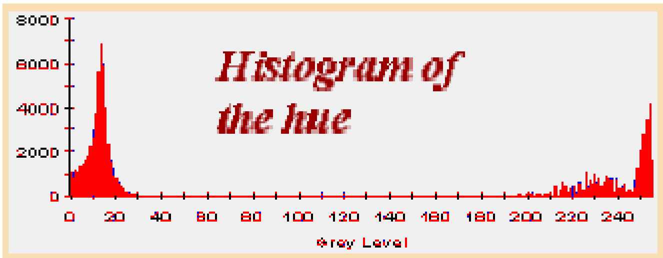
$$(\mathbf{incr}\ h)(x) = \vee[\{|h(x) \div h(y)|,\ y \in \mathbf{B}_r(x)\}] / \mathbf{r}$$

and in the digital **2-D** space, to the ***digital circular gradient:***

$$\alpha(h) = [(\delta_r(h) \div h) \vee (h \div \varepsilon_r(h))] / \mathbf{r}$$

which is ***invariant under rotation*** on the unit circle.

a) Original image



b) Hue band *c) Ordinary hue gradient* *d) Circularly hue gradient*

4.5. Synthetic numerical axis

In case of ***HLS*** representation of color image : all operators combining L, s and the ***increments*** $|h(x) \div h(y)|$, $x,y \in \mathbf{E}$ are independent of the origin of the hue.

Synthetic gradient definition: If we take the two gradients

$$\alpha(h) = (\delta(h) \div h) \vee (h \div \varepsilon(h))$$

$$\alpha(L) = (\delta(L) - L) \vee (L - \varepsilon(L))$$

of the ***luminance*** and the ***hue***, and weight their barycentre by the saturation ***s*** we obtain the ***synthetic gradient***:

$$\beta = s \cdot \alpha(h) + (1 - s) \cdot \alpha(L) \qquad 0 < s < 1$$

The synthetic gradient mixes the hue variations with those of the luminance

a) Rotation invariant hue gradient ***b) Saturation weighted synthetic gradient*** ***c) Gradient of the luminance***

4.6. Segmentations by gradient watersheds and waterfalls

The purpose of this section is to segment color images following the planning:

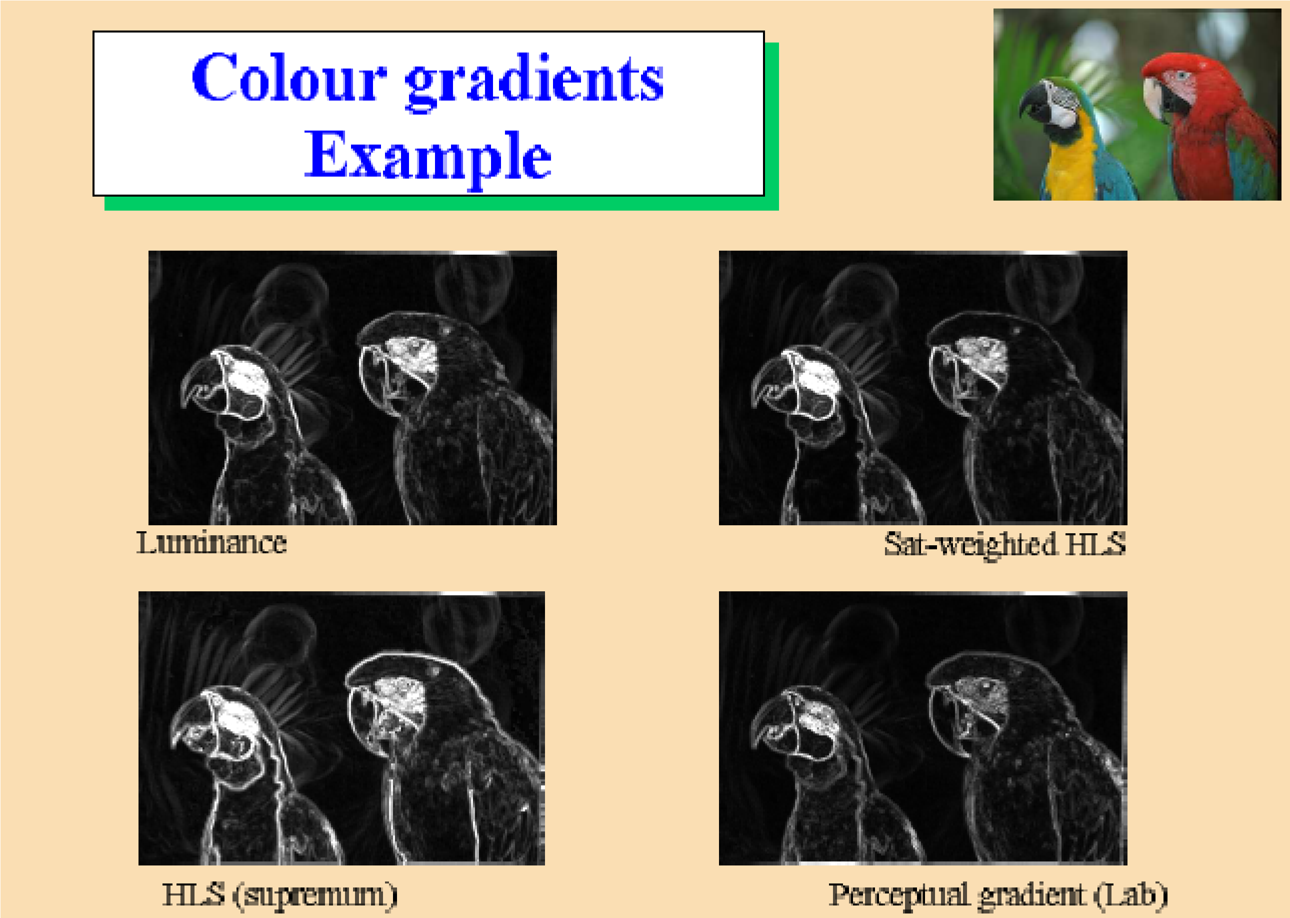
1. Use the conic ***HLS*** space (because prove to be more consistent)
2. Compare the watersheds of four gradients
3. Watershed lines of a gradient
   a. build a scalar gradient of the color image
   b. possibly filter the gradient function
   c. compute the gradient watershed
   d. pursue by a pyramid of watersheds
4. Compare three saturations for the best gradient

The module of the gradient for the color function f at pixel x, $\nabla f(x)$, combines differences between the color at point x and in its unit neighborhood $\mathbf{K}(x)$.

The definitions for the gradient module used here are the following:

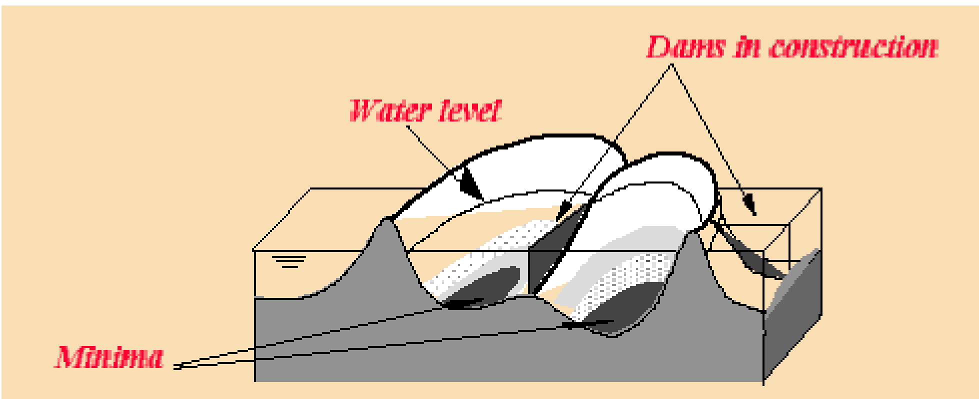
1. ***Luminance Euclidean gradient***
   $\nabla^L f(x) = \nabla_L f(x)$
2. ***Color sat-weighted gradient in conic HLS***
   $\nabla^S f(x) = f_S \times \nabla_c f_H(x) + (f_S^c) \times \nabla f_L(x)$
3. ***Color sup gradient in conic HLS***
   $\nabla^{sup} f(x) = \vee [\nabla_c f_H(x), \nabla f_L(x), \nabla f_S(x)]$
4. ***Euclidean gradient in Lab***
   $\nabla^P f(x) = \nabla_E (f_L, f_a, f_b)(x)$

Examples



Watershed connection algorithm:

1. Suppose that holes are made in each local minimum and that the surface is flooded from these holes.
2. Progressively, the water level will increase.
3. In order to prevent the merging of water coming from two different holes, a dam is progressively built at each contact point.
4. At the end, the union of all complete dams generates the ***watersheds***.

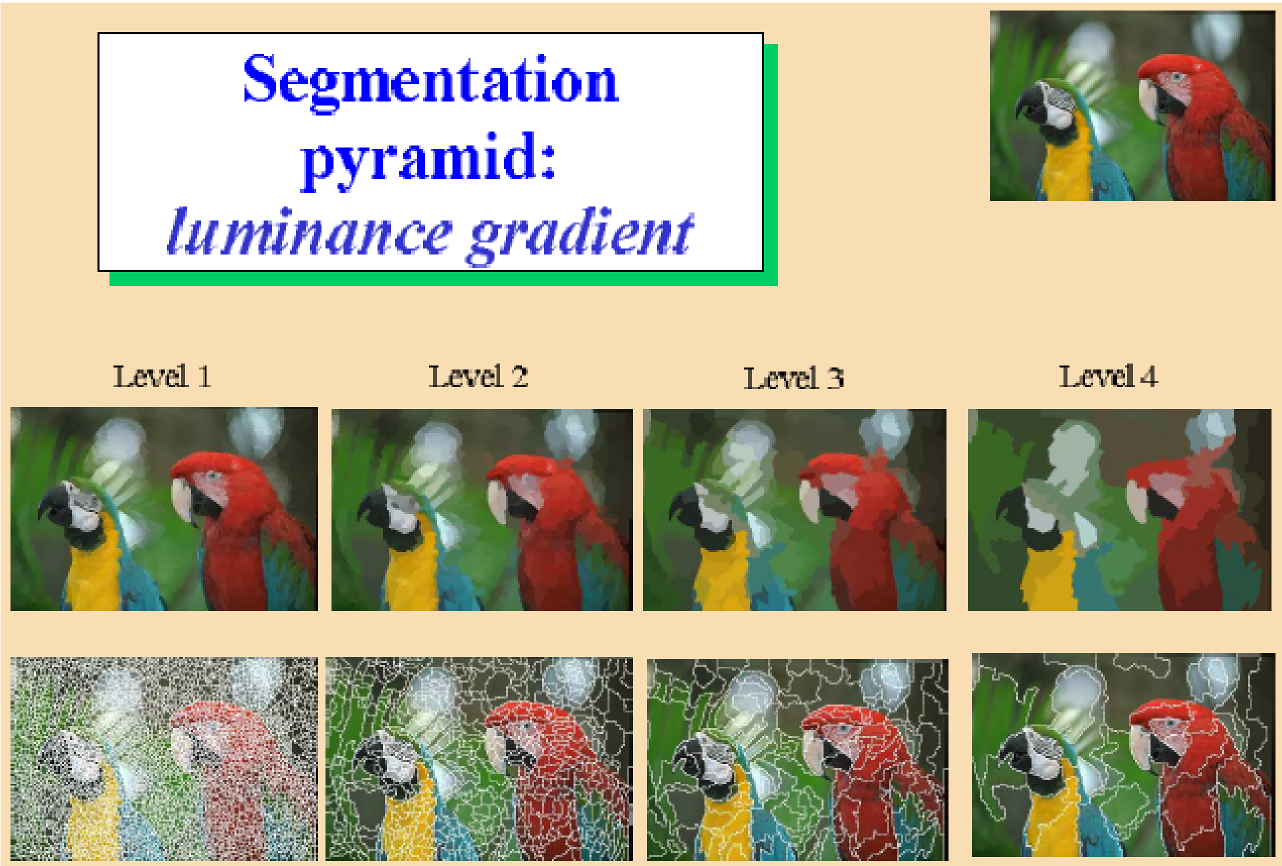


4.7. Hierarchical segmentation

The aim of ***image segmentation*** is partitioning images into disjoint regions whose contents are homogenous in color, texture.

Multiscale segmentation:

- The partitions family is composed of a hierarchical pyramid with increasing partitions.
- Here we use the waterfall algorithm, i.e. a non-parametric pyramid of watersheds, comparing different color gradients
- This approach involves a color representation based on ***hue***, ***brightness***, ***saturation***, where the saturation component plays an important role for merging both chromatic and achromatic information.

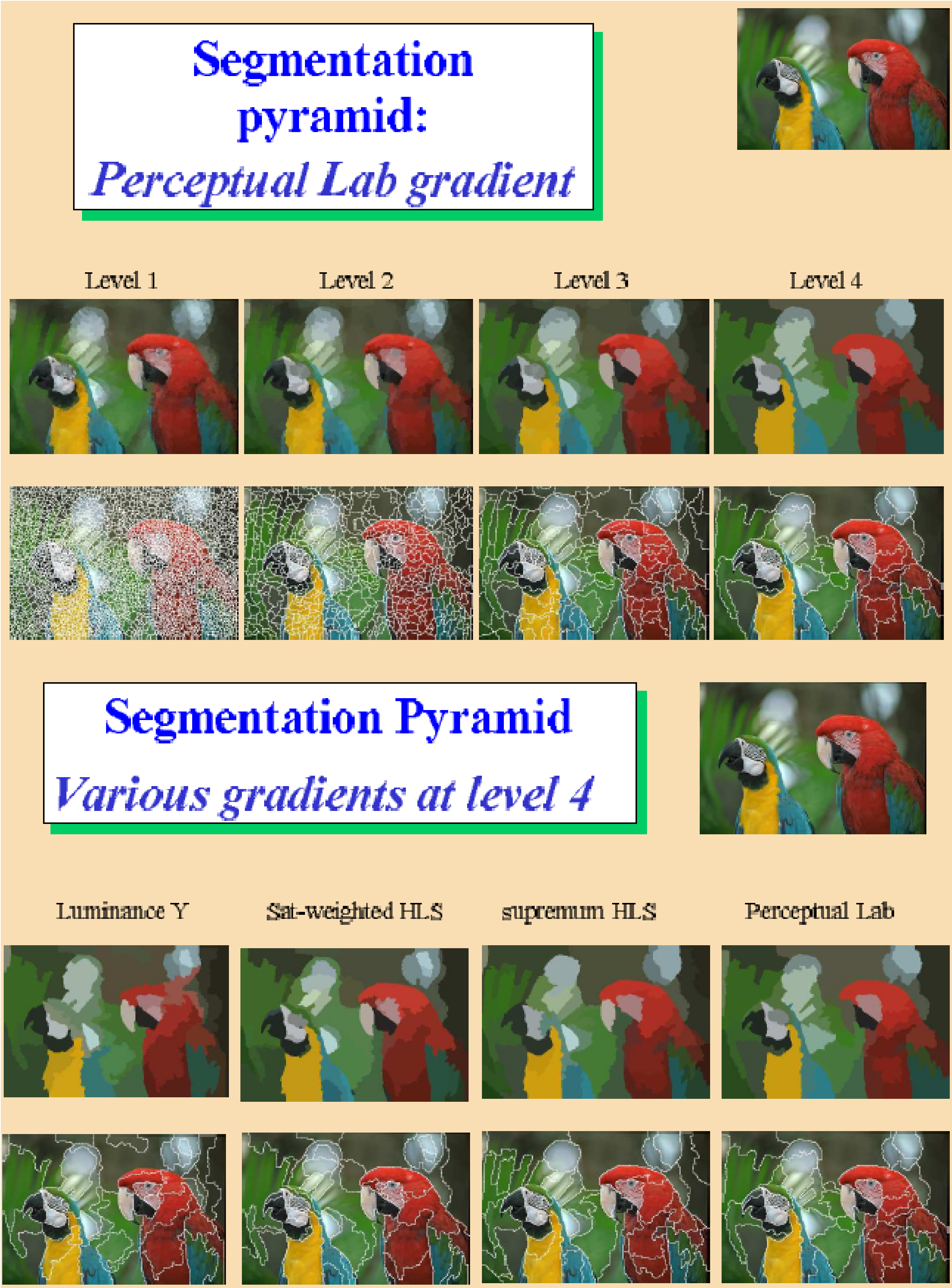


Segmentation pyramid: *HLS sat-weighted gradient*

Level 1 | Level 2 | Level 3 | Level 4

Segmentation pyramid: *HLS gradient (by sup)*

Level 1 | Level 2 | Level 3 | Level 4



Discussion

- The luminance alone (∇^L) generates poor segmentations.

- Visually, the most contrasted gradient is ***HLS***-supremum ∇^{sup}, it also yields better segmentations than the only luminance ∇^{L} , as well as the perceptual ∇^{P}.

- Finally, the best partitions are obtained by using the ***saturation weighted gradient***. However, they still are over-segmented.

- To which extend does the choice of the saturation determine the quality of the segmentation?

4.8. Two-levels mixed segmentation

The goal of this application is to segment the head and the bust.
The following ***color/shape segmentation*** algorithm, proposed by ***Ch. Gomila***, is a two-levels mixed segmentation:

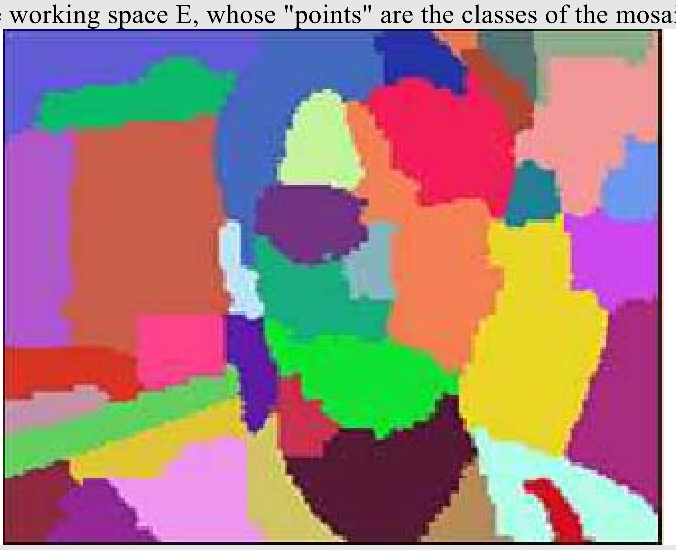
1. The image under study is given in the standard color video representation ***YUV:***

Y = 0.299r'+0.587g'+0.114b'

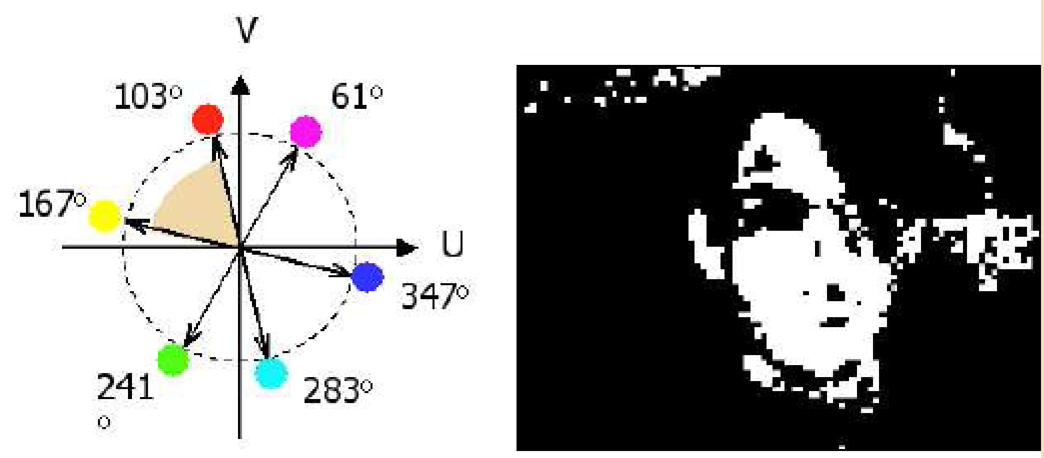
U = 0.492(b'-y')

V=0.877(r'-y').

2. A previous segmentation resulted in the tessellation depicted here in false color. For the further steps, this mosaic becomes the working space E, whose "points" are the classes of the mosaic;



3. Classical studies have demonstrated that, ***for all types of human skins***, two chrominances ***U*** and ***V*** practically lie in the sector region depicted in a). By thresholding the initial image by this sector, we obtain the set b), whose a small filtering by size suppresses the small regions, yielding a marker set;



a) *b)*

4. ***Segmentation by color:*** All "points" of **E** that contain at least a pixel of the marker set, or of its symmetrical w.r.t. a vertical axis are kept, and the others are removed: this produces the opening $\gamma_1(\mathbf{E})$ depicted in b) ;

a) *b)*

5. ***Segmentation by shape:*** For the bust, an outside shape marker made of three superimposed rectangles is introduced. All their pixels that belong to a "point" of $\gamma_1(\mathbf{E})$ are removed from the bust marker, since this second marker must hold on $\mathbf{E} \setminus \gamma_1(\mathbf{E})$ only. That is depicted in b), where one can notice how much the upper

a) *b)* *c)*

6. ***Final result:*** The union $\gamma_1(\mathbf{E})$ Y $\gamma_1[\mathbf{E} \setminus \gamma_1(\mathbf{E})]$ defines the zone inside which the initial image is kept, as depicted in b)

a) b)

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